A Little Antenna Math & Other Related Topics
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Frequency and Wavelength

The speed of all electromagnetic radiation or waves, whether light or radio frequency (RF) energy, is about 671 million miles per hour – pretty fast. "Miles per hour" is something we can relate to more easily than "meters per second" but it is not a very convenient unit of measure for doing antenna math. So in the interest of real science let's convert this number to "meters per second". First we convert to "miles per second" by doing some division and multiplication and we get the following:

671,000,000 miles per hour / (60 seconds per minute * 60 minutes per hour) =

671,000,000 miles per hour / 3,600 seconds per hour ≈ 186,000 miles per second
(≈ means 'is approximately').[1]

Now that's a familiar number from high school science! The Sun is about 93 million miles from Earth so if it suddenly extinguishes then things will get very dark around here about 500 seconds or 8⅓ minutes later. Let's continue by converting miles to meters. One meter is 39.37 inches long or about 3¼ feet. With some more multiplying and dividing we have:

5,280 feet per mile * 12 inches per foot = 63,360 inches per mile

or

63,360 inches per mile / 39.37 inches per meter ≈ 1,609.35 meters per mile.

So the speed of light is approximately:

186,000 miles per second * 1,609.35 meters per mile ≈ 299,339,100 meters per second.

For all practical purposes this is 300 million meters per second (m/s). It turns out that in 1983 the speed of light in a vacuum was defined to be exactly 299,792,458 m/s. This *constant* is represented in the following formulas by the italic letter *c*.[2]

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[1] The symbol * means "multiplied by". Sometimes this is represented with a dot. And more often than not the symbol is omitted altogether when variables or constants are multiplied, as in the next footnote.

[2] http://en.wikipedia.org/wiki/Speed_of_light. Geek alert: in a vacuum $c_o = 1 / \sqrt{\mu_o 0_o}$ where μ_o is the *magnetic* constant or "permeability of free space" (exactly $4\pi \times 10^{-7}$ henries per meter), and 0_o is the *electric* constant or "permittivity of free space" (approximately 8.85419×10^{-12} farads per meter). "Free space" is a place far, far away in a perfect vacuum with nothing around to interact; very theoretical.

It so happens that

$$c = f * \lambda \qquad (\text{or } f = c / \lambda)$$

or

$$300{,}000{,}000 = f * \lambda$$

where f is the frequency in hertz (or cycles per second) and λ (the Greek letter lambda) is the wavelength in meters. Just accept these formulas as gospel. Even though they look simple they are the basis for some very hard-core physics.[3] f and λ are called *variables* since they can each be assigned different numeric values. If we express the frequency f in *mega*hertz (or millions of hertz) instead of in hertz, the formula becomes:

$$300 = f * \lambda$$

Since c is always constant we now have a relationship that says the following: if the frequency goes up (*i.e.*, becomes higher) then the wavelength *must* go down (*i.e.*, becomes shorter) and vice versa. Table 1 shows the popular amateur radio frequency allocations and their corresponding "band" (which is the wavelength or λ) demonstrating this relationship.

Table 1 – The Relationship of Frequency and Wavelength

Frequency (f in megahertz)	**Band (λ in meters)**	**$f * \lambda$**
1.800 – 2.000	160	288 - 320
3.500 – 3.600	80[4]	280 – 288
3.600 – 4.000	75	270 - 300
5.330 – 5.405	60	319.8 - 324.3
7.000 – 7.300	40	280 - 292
10.100 – 10.150	30	303 - 304.5
14.000 – 14.350	20	280 - 287
18.068 – 18.168	17	307.2 - 308.9
21.000 – 21.450	15	315 - 321.8
24.890 – 24.990	12	298.7 - 299.9
28.000 – 29.700	10	280 - 297
50.000 – 54.000	6	300 - 324
144.00 – 148.00	2	288 - 296
222.00 – 225.00	1.25	277.5 - 281.25
420.00 – 450.00	0.70	294 - 315

[3] Geek alert: speaking of "simple" formulas, it's also the case that $f = E / h$ where E represents photon energy and h is the Planck proportionality constant. This formula is known as the Planck-Einstein equation, see http://en.wikipedia.org/wiki/Planck_constant. Anything with Einstein's name attached to it is serious stuff!

[4] The Morse code (or CW) subband of 3.500-4.000 MHz is traditionally known as 80 meters. The phone (LSB or AM) subband of 3.500-4.000 MHz is traditionally known as 75 meters. The boundary between the two moves around occasionally; it's currently at 3.600 MHz.

Notice how the range of values in the third column are all near '300' but, with a couple of exceptions, don't actually include '300'. That's because our amateur band 'names' are *conventions* based on the range of frequencies shown in the left column above. Our 60 meter band should probably be the '56' meter band and our 15 meter band should probably be the '14' meter band so that values of $f * \lambda$ are closer to 300. Similarly for the 1¼ meter band. I'm not sure about the naming of our 60 meter band since there is also a 60 meter shortwave international broadcast band from 4.750 to 5.060 megahertz. For our 21.000-21.450 MHz band, my guess is that '15 meters' was chosen so as not to confuse 14 meters and 14 megahertz (which is in our 20 meter band). But wait! There's a 15 meter shortwave band at 18.900-19.020 MHz. Confused? Me too! The amateur bands above 225 MHz are shorter than one meter so in the case of the 0.70 meter band we'll refer to it as the 70 *centi*meter or 70 cm band.

Resonance, Impedance, and Reactance

Resonance, impedance, and reactance are three concepts that cause amateur radio operators a lot of confusion. Let's start with *resonance*. Your radio's antenna is actually an alternating-current (AC) circuit.[5] In general, it is either a *series LC* or a *parallel LC* tuned circuit where *L* stands for inductance and *C* stands for capacitance.

When your antenna is resonant, or "in resonance", then *L* and *C* take on values that cancel each other electrically.[6] When this happens the current and voltage travelling along your antenna are exactly "in phase". So if they cancel each other, what's left? Just the resistive component that we'll refer to below as *R*. This resistive component is called *impedance* and is measured in ohms (represented by the Greek letter omega, Ω). Impedance is the opposition to the flow of current in an AC circuit. It turns out that in free space a lossless ½ wavelength dipole has a feedpoint impedance $Z \approx 73\ \Omega$. A lossless ¼ wavelength vertical has a feedpoint impedance $Z \approx 37\ \Omega$.[7] It is no coincidence that 37 is about half of 73.

What happens if your antenna is not in free space,[8] but rather in a more typical location such as your backyard or on the roof of your car? Then there are probably "reactive" components to consider. It turns out that these three concepts, resonance, impedance, and reactance, can be described together with the formula:

$$Z = R + jX$$

where X is the *reactance*, also measured in ohms. In the field of mathematics the symbol j represents the "imaginary number," *i.e.*, the square root of negative 1.[9] It's imaginary because

[5] Direct-current (DC) circuits also have inductive and capacitive components but do not exhibit reactance.
[6] This is not necessarily, and rarely occurs, at the point where the *standing wave ratio* (or SWR) is 1:1. See W2DU's book "Reflections III" for a *very* detailed explanation.
[7] A little more precisely, these values are $73.14 + j42.50$ and $36.57 + j21.25$ ohms, respectively.
[8] Last geek alert: Using the same constants from footnote #2, the *impedance of free space* is $\sqrt{\mu_o / 0_o} \approx 376.73\ \Omega \approx 120\pi\ \Omega \approx \mu_o c_o$. An antenna is actually a "transformer" that transforms, *at resonance*, the impedance of free space to the feedpoint impedance. The impedance of free space is also known as intrinsic impedance.
[9] http://en.wikipedia.org/wiki/Imaginary_number

the "rules" of mathematics for real numbers say that multiplying two negative numbers together results in a positive number. Instead, here we have $j = \sqrt{-1}$ so that $j * j = j^2 = -1$. This is one of those concepts that you just have to accept unless you want to dive into some serious math called phasor (or vector) analysis. We use the symbol j for antenna and other electrical engineering math instead of the traditional symbol i since we're already using I to represent current.

In the preceding formula the *real* part of the antenna impedance, *R*, represents power that is either radiated away or absorbed within the antenna. The *imaginary* part of the impedance, *jX*, represents power that is stored in the "near field" of the antenna. This is non-radiated power. If there are no reactive components to your antenna at a given frequency then the reactance equals zero and we have:

$$Z = R + 0 \quad \text{or} \quad Z = R$$

and in this case the impedance Z is resistive only. In general, your antenna will be most efficient at this frequency. However, if jX is a positive number (> 0) then the reactance is said to be "inductive" (*e.g.*, see footnote #7 above). On the other hand, if jX is negative (< 0) then the reactance is said to be "capacitive".[10] So reactance is the opposition to the flow of current in an AC circuit caused by inductance or capacitance. An antenna that is physically short for a given frequency f exhibits capacitive reactance. To make a "physically short" antenna look "electrically longer" we can add inductance to cancel this capacitance and reduce the reactance closer to zero. This is the coil (an inductor) that you see at the base of some mobile antennas, particularly those for the high frequency (HF) bands below 30 MHz.

Applying this to Antennas

Antennas come in two basic varieties, Hertz and Marconi, named after their 19th century inventors.[11,12] Hertz antennas are self-resonant and thus independent of the radio frequency (RF) "ground". They include popular designs like the half wave (½ λ) dipole and the Yagi-Uda array .[13] Marconi antennas always require a counterpoise or some other type of RF ground to provide a "return current" path. These include monopole designs like the quarter wave (¼ λ) vertical. Generally speaking, for any particular wavelength an ungrounded ½ λ antenna is resonant at the same frequency as a grounded ¼ λ antenna.

The formula for the resonant length of a ½ λ dipole is:

$$468 / f = \text{length in feet}$$

and the formula for the resonant length of a ¼ λ vertical is half of that, or:

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[10] When voltage (E) and current (I) are *not* in phase, voltage leads current in an inductive (L) circuit and current leads voltage in a capacitive (C) circuit. The mnemonic "ELI the ICE man" is useful for recalling this relationship.

[11] http://en.wikipedia.org/wiki/Heinrich_Hertz

[12] http://en.wikipedia.org/wiki/Guglielmo_Marconi

[13] http://en.wikipedia.org/wiki/Yagi-Uda_antenna

$234 / f$ = length in feet

where f is the frequency in megahertz. Ok - earlier we converted feet to meters and now we're talking about feet again - what gives? Before we were doing real science; now we're just being practical. The number '234' is easy to derive since it is half of 468, but where did '468' come from? From our earlier discussion we discovered there are about 3¼ feet per meter, and more accurately 3.28 feet per meter. So keeping the math compatible with megahertz we have from the top of page 2 that:

$300 = f * \lambda$ or $300 / f = \lambda$ (where λ is in meters)

so that:

$(300 * 3.28) / f = \lambda$ or

$984 / f = \lambda$ (where λ is now in feet).

Dividing both sides of this formula by 2 we have:

$492 / f = \lambda / 2$ = length in feet of half of a wavelength at a frequency f.

The astute reader will quickly observe that 492 is not quite the same number as 468! Actually 492 would work if our antenna was in free space but that isn't going to happen anywhere around here. So a rule of thumb is to shorten the antenna by 5 percent to compensate for various things like the velocity factor of the radiator (*i.e.*, the wire), capacitive end effects, proximity effects, and the diameter of the radiator.[14] Taking 95 percent of 492 we have:

$(492 * 0.95) / f \approx 468 / f$ = a more practical length in feet of a ½ λ dipole.

Table 2 – Antenna Lengths in Feet

Frequency in MHz	**Band (in meters)**	***½ wavelength***	***¼ wavelength***
1.800	160	260'	130'
3.500	80	134'	67'
5.330	60	88'	44'
7.000	40	67'	34'
10.100	30	47'	24'
14.000	20	34'	17'
18.068	17	26'	13'
21.000	15	23'	12'
24.890	12	19'	10'
28.000	10	17'	8'5"
50.000	6	9'4"	4'8"
144.000	2	39"	20"
222.000	1.25	26"	13"

[14] The input impedance of a dipole is slightly inductive at exactly 0.500 λ, see footnote #7 above. The reactive part of the input impedance goes to zero at 0.475 λ. So we have $0.475 * 2 = 0.95$ or 95%.

420.000	0.70	13.5”	7”

Ward Silver, NØAX provides a nice history of this magical number and points out that 468 is *rarely* correct.[15] What to do then? Just use 468 or its ¼ λ equivalent of 234 as a *starting* point for the resonant length measured in feet.

Table 2, above, provides starting points for the amateur bands from 160m through 70cm rounded *up* to the next foot (except above 12m where it helps to be a little more precise). These starting points are based on the *lowest* frequency in the band. We round up since it’s far easier to trim wire than to add wire.[16] Get the length close and let your antenna tuner take care of the unknowns. And if it has been a while since you’ve used a calculator it might be useful to replicate the numbers in the two tables above; it’s good practice for your license exam!

Quarter-wavelength vertical antennas are commonly used in mobile installations and on the amateur bands below 20 meters. The “image antenna”, or the other half, of your ¼ wavelength vertical is created by the antenna’s counterpoise system. This counterpoise system can consist of one or more ¼ wavelength wires that extend radially (hence the name ‘radials’) from the base of your vertical radiator.[17] They are cut for the lowest band that your antenna will operate on, and more are always better than fewer. But don’t overdo it - there are diminishing returns after 32 radials, and the radials can be shorter than ¼ wavelength, if necessary, to fit the available space (they’re detuned in close proximity to the ground anyway). If the radiator is mounted on the ground then the radials lay on or, better yet, just under your lawn. See the footnote below for an *easy* way to do this without digging up your lawn and getting the XYL overly excited.[18] If your vertical is elevated (*e.g.*, mounted on your roof) then you can get by with far fewer, usually two to four, but they *must* be ¼ wavelength long for *each* band you'll operate on. When cutting ¼ wavelength radials do *not* adjust for capacitive end effects, etc. Use the free space formula of 246 / *f* (instead of 234 / *f*) for the length in feet of your radials.

Getting to the Antenna

The antenna *system* begins right at the 50Ω output impedance of the transmitter.[19] The antenna system includes the feedline, or transmission line. There are antenna systems that don’t have a feedline. These are ½ wavelength, or longer, lengths of wire and the transmitter is typically being operated in a portable setting, *e.g.*, at Field Day or while camping. Matching the transmitter’s 50Ω output impedance to the antenna’s feedpoint impedance can be a little tricky and will usually require an automatic antenna tuner (or ATU) with a wide matching range, typically 10:1 SWR. But in most cases, particularly where your transmitter or transceiver is located indoors, you’ll need to get from the radio to the antenna via a transmission line.

[15] http://www.eham.net/articles/23802

[16] The ARRL Antenna Book suggests starting with 485/*f* or 490/*f* for a dipole’s length in feet (Chapter 9.1.1), *i.e.*, don’t apply the 5 percent compensating factor discussed earlier.

[17] The counterpoise system might also be the metal roof of your car. In this case we refer to it as a ground plane.

[18] https://www.dxengineering.com/techarticles/verticalantennainfo/how-to-put-amateur-radio-radial-wires-down-without-digging

[19] Technically the system is *everything* attached via a common ground but let’s try to keep this discussion manageable.

Transmission lines for frequencies below microwave are of two basic types: coaxial cable and parallel line. The most popular varieties of parallel line have a characteristic (or surge) impedance Z_o of 300Ω or 450Ω.[20] Parallel transmission line has very low loss compared to most coaxial cable but new hams shy away from it because its lack of shielding can cause RF feedback problems if not managed properly. On the other hand, coaxial cable is self-shielded making RF feedback problems much more manageable, and typically has a characteristic impedance Z_o of 50Ω just like the output impedance of your transmitter. By the way, what's so special about 50Ω? Why not 75Ω since RG-6 Cable TV coaxial transmission line is so readily available? 50Ω just happens to be very close to the average (both the geometric mean and the arithmetic mean) of 30Ω where maximum power is transferred and 77Ω where minimum attenuation of an incoming RF signal occurs.[21] In other words, it's a nice compromise and is an RF industry convention. However, RG-6 makes a great feedline for a receive-only system.

Some good websites and books about antennas

Here is a mix of both practical and theoretical, free and pricey:
http://www.antenna-theory.com/
http://www.w8ji.com/antennas.htm
http://www.k0bg.com/
http://www.aa5tb.com/antennas.html
http://www.arrl.org/how-antennas-work
ON4UN's book "Low-Band DXing, 5th Edition":
    https://home.arrl.org/action/Store/Product-Details/productId/134491
ARRL's "Antenna Book, 24th Edition":
    https://home.arrl.org/action/Store/Product-Details/productId/114288
W2DU's book "Reflections III":
    http://store.cq-amateur-radio.com/shop/reflections-iii/
Radio Society of Great Britain's (RSGB) book "Stealth Antennas":
    https://www.rsgbshop.org/acatalog/Stealth-Antennas-1734.html
W8JK's book "Antennas, 3rd Edition":
    http://www.amazon.com/Antennas-John-D-Kraus/dp/007123201X

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Latest version of this document is at http://www.n8ik.net/classes.php

[20] The "characteristic impedance" of a transmission line is $Z_o = \sqrt{L / C}$ where L and C are measured in henries and farads, respectively, per unit of length. Note the similarity of this formula to the formula in footnote #8.
[21] http://www.microwaves101.com/encyclopedias/why-fifty-ohms